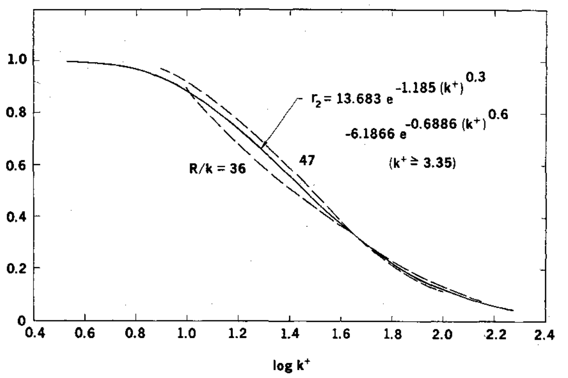

Fig. 5 The implied form of r_2 vs logk^+ according to Spangler's data for $R/k = 47$ and 36, together with a fitted exponential form.

Figure 3 reproduces Spangler's data[3] on the effects of uniform machined roughness on pipe flow of a 31 wppm solution of polyacrylamide P-295, as plotted by Poreh.[2] The assignment of the α value of 17.2, the v_{*crit} of 0.0968 fps, and the equivalent sand-grain roughness heights are explained in Ref. 3. The dashed curves through the data for $R/k = 36$ and 47 are our fairings (R is pipe radius). We consider the data for $R/k = 18$ to allow the interpretation that the behavior in that case is Newtonian.

The implied forms of r_1 and r_2 vs k^+ have been inferred from the data in Fig. 3 using Eq. (6) with (7) or (8), for the values of kv_{*crit}/ν determined from Fig. 7b of Ref. 3. The results are shown in Figs. 4 and 5; the dashed curves correspond to the faired curves in Fig. 3.

The results for the $\Delta B_m'$ model of Eq. (8) are much more promising than those for the model of Eq. (7). An exponential form has been fitted to the fairly coincident data curves, as shown in Fig. 5, assuming that $r_2 = 1.0$ for $k^+ \leq 3.35$.

The resultant behavior of ΔB according to Eqs. (6) and (8) for the case of $\alpha = 17.2$ is shown in Fig. 6, which should be compared with Fig. 2. The new model displays a much smoother onset of roughness effect, appreciably less roughness degradation of ΔB for small values of kv_{*crit}/ν, and the Newtonian-fluid value of k^+ at roughness effect onset, independent of α and v_{*crit}. It also displays a polymer effect cutoff at high kv_{*crit}/ν, due to r_2 keeping $r_2 v_*/v_{*crit} < 1$ for all k^+, which is consistent with our interpretation of the data for $R/k = 18$ in Fig. 3. Of course, we must emphasize that only further work can determine if the model is really useful; if r_2 depends on polymer type, concentration, and type of roughness; and how the onset of roughness effect depends on k^+.

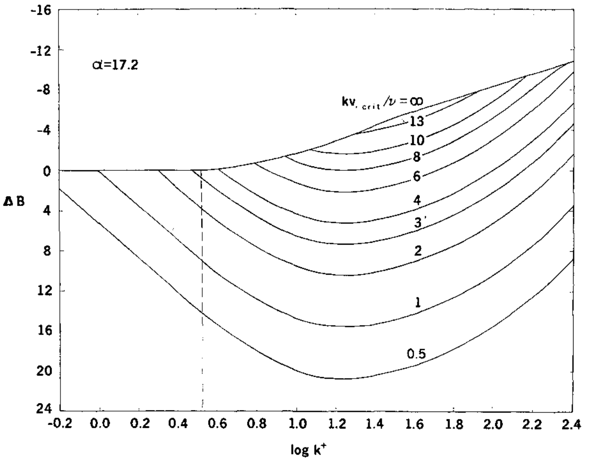

Fig. 6 ΔB vs logk^+ according to the fitted exponential form for r_2 for various values of kv_{*crit}/ν, for $\alpha = 17.2$.

Conclusions

Poreh's proposed model for the log law ΔB shift due to polymeric friction reduction with wall roughness contains an anomaly for low values of kv_{*crit}/ν. The peculiarly abrupt onset of roughness effect arises from an otherwise insignificant detail in his model for the Newtonian-fluid case. Some other possible models which display a smooth onset of roughness effect are considered. One of the two models tested with and fitted to Spangler's data seems to merit further investigation. It implies much less degradation of ΔB for small values of kv_{*crit}/ν than Poreh's model.

References

[1] Poreh, M., "Flow of Dilute Polymer Solutions in Rough Pipes," *Journal of Hydronautics*, Vol. 4, No. 4, Oct. 1970, pp. 151–155.

[2] Meyer, W. A., "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Non-Newtonian Fluids in Pipes," *American Institute Chemical Engineers Journal*, Vol. 12, No. 3, May 1966, pp. 522–5.

[3] Spangler, J. G., "Studies of Viscous Drag Reduction with Polymer Including Turbulent Measurements and Roughness Effects," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, New York, 1969, p. 131.

Reply by Author to A. G. Fabula and D. M. Nelson

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The technical comment by Fabula and Nelson on the "Flow of Dilute Polymer Solutions in Rough Pipes"[1] discusses some limitations of the model proposed in Ref. 1 and presents an attempt to describe the available data by different mathematical expressions.

I fully agree with the comment that the function $p(k/\delta)$ chosen to approximate analytically the average data of Nikuradse changes abruptly in the neighborhood of k/δ =

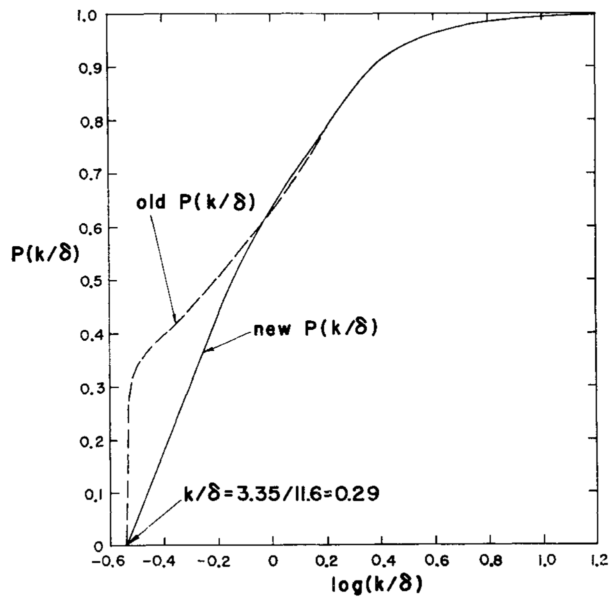

Fig. 1 Plot of $p(k/\delta)$ vs log(k/δ).

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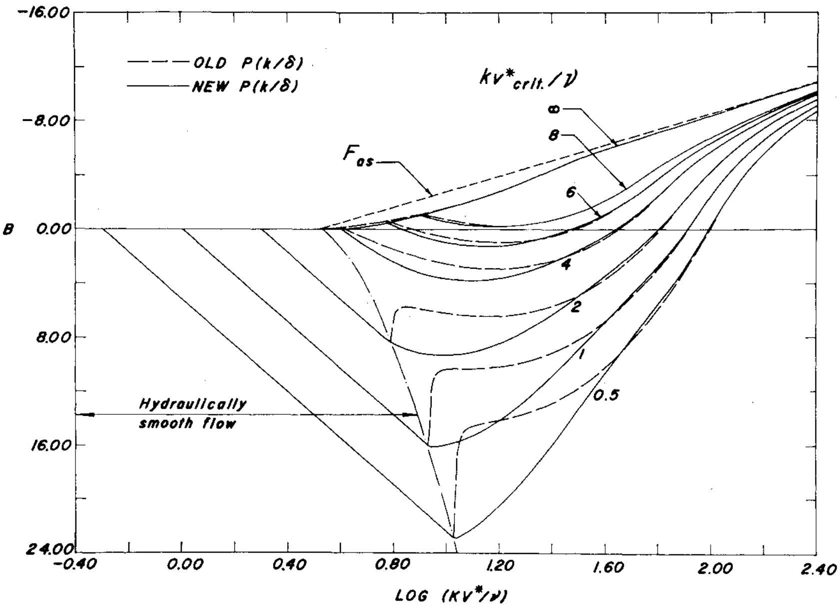

Fig. 2 **ΔB vs kv^*/ν for the old and new $p(k/\delta)$; (α = 17.2).**

0.29. This has already been recognized by the author in a previous study.[2] To overcome this deficiency it is suggested that the old function $p(k/\delta)$ in the region $0 < k/\delta < 20/11.63$ be replaced by a new approximation:

$$p(k/\delta) = [1 - \mathrm{EXP}\{-0.3(w - 3.35)\}](0.53 + 0.014w)$$

where $w = 11.63\, k/\delta$. The new and the old functions are compared in Fig. 1.

The effect of the change on the shift ΔB for various values of kv^*_{crit}/ν is shown in Fig. 2. One sees that for $kv^*_{\mathrm{crit}}/\nu >$ 4.0, where experimental data are available, the effect of the change on ΔB is rather small. The effect of the change on smaller values of kv^*_{crit}/ν is to smoothen the discontinuity at the onset of the roughness effect.

I also agree with the comment that the shift ΔB at small values of kv^*_{crit}/ν in the proposed model is very sensitive to the details of the roughness expressed by the function p. This is indeed an unfortunate feature for those trying to predict the effect of the roughness on the flow of dilute polymer solutions on the basis of measurements without polymers. The opposite task would certainly be much easier. I disagree, however, with the conclusion that one should look for a less sensitive function of k/δ, unless it rests on some physical arguments or experimental evidence which suggest that the details of the roughness cannot have such a strong effect on the flow. Those were missing from the comment.

The available data, it appears to the author, do suggest that small changes in the details of the roughness cause large changes in the value of ΔB in the range $kv^*_{\mathrm{crit}}/\nu < 4.0$. Consider, for example, the measured effect of the roughness $k/R = 0.028$ on the friction coefficient with and without polymers (Ref. 1, Figs. 5 and 1) as summarized in Table 1. The friction coefficient is a function of the relative roughness and the Reynolds number. It is very clear that both the effect of roughness and the Reynolds number at high speeds are much larger in the presence of polymer additives. This large dependency on the value of k suggests that ΔB would also be sensitive to the value of $p(k/\delta)$, which describes the true hydrodynamic effect of a particular surface. (By reducing the dependence of ΔB on k^+, Fabula and Nelson had increased the dependence of ΔB on kv^*_{crit}/ν at large values of k^+. Compare, for example, the effect of a change from $kv^*_{\mathrm{crit}}/\nu = 4$ to $kv^*_{\mathrm{crit}}/\nu = 10$ at $k^+ = 1.6$ in Figs. 2 and 6.

Table 1 **Effect of roughness k/R = 0.028 on friction coefficient with and without polymers**

	$k/R = 0$		$k/R = 0.028$		% Increase
Reynolds number	10^5	2×10^5	10^5	2×10^5	2×10^5
Flow without polymers	0.018	0.015	0.043	0.046	300
Flow with polymers	0.082	0.06	0.027	0.038	630

The proposed functional form (with r_2) also differs from the original model in its prediction of the onset of the roughness effect. The absolute value of surface roughness in turbulent flow of Newtonian fluids does not indicate whether a wall is rough or not. It is the relative roughness k/δ, or equivalently k^+, which determines the effect of the roughness. A particular pipe can be hydraulically smooth in one flow and rough in another flow with a smaller viscous sublayer. Since polymers increase the thickness of the viscous sublayer, it is quite plausible that they will also increase the maximum upper size value of roughness that does not effect the drag. This feature is maintained in the model proposed in Ref. 1 and is missing in the description proposed by Fabula and Nelson.

The analysis proposed in Ref. 1 is based on a simplified physical model which assumes a similarity between the effect of the relative roughness in flows with and without polymers. It does not contain any new experimental coefficients that have to be determined by measurements in flows of polymer solutions in rough pipes. As shown, the model does describe, at least qualitatively, the available experimental results, and one should not expect more from such a model. A more reliable description of the effect of roughness on drag reduction might be obtained only on the basis of a rigorous theory and many more experiments.

References

[1] Poreh, M., "Flow of Dilute Polymer Solutions in Rough Pipes," *Journal of Hydronautics*, Vol. 4, No. 4, Oct. 1970, pp. 151–155.

[2] Poreh, M., "Drag Reduction of Flat Plats with Surface Roughness," Technical publication 222 (in print), Naval Undersea Research and Development Center, Pasadena, Calif.